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Kinetic models of plasma–particle charge, momentum and energy transfer under rarefied flow conditions

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Abstract: The methods of kinetic theory of rarefied ionized gases which have been applied to the analysis of plasma-particle interaction are discussed. The main attention is paid to the effects of particle charging, Debye screening and influence of plasma electrons and ions on the intensity of charge, momentum and energy transfer. The models of stationary plasma with Maxwell, monoenergetic or cold ions and also subsonic and hypersonic plasma flows are described.

INTRODUCTION

One of the most distinguishing features of plasma action on particles of materials is associated with gas ionization and presence in the flow not only neutral molecules but also negative electrons and positive ions which undergo collisions with particle surface. This results in the effect of particle charging (1-19), i.e. accumulation of charges on a particle and set-up of local electrostatic field influencing the motion of electrons and ions. In general case, charge, momentum and energy flows depend on spatial electrostatic potential distribution around the particle.

Kinetic theory description of plasma-particle interaction consists in a combined solution of the Boltzman equation for the velocity distribution functions of molecules, electrons and ions, and the Poisson equation for the electrostatic potential. After the distribution functions are found from such an analysis, it becomes possible to calculate any macroscopic characteristic of the plasma, in particular, the fluxes of charge, momentum and energy transferred to the particle by each of the three plasma species.

However, this problem entails great mathematical difficulties, and analytical or even numerical description of the transfer processes is possible only in some special situations under simplifying assumptions.

This paper is concerned with the kinetic methods of analysis, which have been applied to the description of plasma-particle charge, momentum and energy transfer in free-molecular regime ($\mathrm{Kn}_j = l_j / R \gg 1$) for an arbitrary ratio between Debye length r_D and particle radius R.

GENERAL APPROACH AND BASIC EQUATIONS

Since the action on a particle of neutral plasma atoms in free-molecular regime is well studied, the main attention is paid to the transfer of the charged plasma species - electrons and ions.

The kinetic problem is considered in a local (but large enough with respect to the particle size) plasma region surrounding the particle. The basic equations governing the plasma species distributions and electrostatic field around the spherical particle are the collisionless Boltzmann equation

$$\mathbf{v}\frac{\partial f_j}{\partial \mathbf{r}} - \frac{Z_j e}{m_j}\frac{\partial f_j}{\partial \mathbf{r}}\frac{\partial f_j}{\partial \mathbf{v}} = 0 \qquad (1)$$

and the Poisson equation

$$\Delta\varphi = -4\pi e(N_i - N_e) \tag{2}$$

where $Z_a = 0,\ Z_i = 1,\ Z_e = -1$.

The boundary conditions for the velocity distribution functions f_j are specified by the known distributions of the *j*th kind of plasma species far from the particle in nondisturbed flow region and by the given law of reflection from the particle surface. The electrostatic potential assumes zero value in the plasma region far from the particle. The value of the potential at the particle surface φ_f (floating potential) is determined proceeding from the condition of equality of the fluxes of electrons and ions absorbed by the particle. For metallic (conducting) particle,

$$I_e^-(\varphi_f) = I_i^-(\varphi_f) \tag{3}$$

For nonmetallic (nonconducting) particle, since the potential distribution over its surface is nonuniform, a similar equation must be solved at every point of the particle surface.

Owing to a considerable difference of the average thermal velocities of electrons and ions, $\bar{v}_e / \bar{v}_i >> 1$, the particle obtains a negative floating potential, such that $-e\varphi_f / kT_{e\infty} > 1$.

The total heat flux transferred to the particle from plasma $Q = Q_a + Q_e + Q_i$ is formed of molecular, ionic, and electronic components

$$Q_a = K_a^- - K_a^+\ ,\quad Q_i = K_i^- + I_i^- W_i - K_i^+\ ,\qquad Q_e = K_e^- + I_e^- W_e \tag{4}$$

and besides the fluxes of kinetic energy of incident and reflected plasma species $K_j^{\mp}$ also includes the energies of charging states of ions $W_i = E_i - \Phi_e$ and electrons $W_e = \Phi_e$ released at every collision with the surface.

It is well known that the drag force acting upon the particle in a plasma flow is conditioned by the momentum transfer by heavy plasma species - molecules and ions. The contribution of electrons to the drag force is negligible. The total drag force $F = F_a + F_i$ is the sum of the terms due to direct collisions with the particle (F_h^s) and reflection from its surface (F_h^r) of molecules and ions, and Coulombic interaction (F_i^C) of noncollected deflected ions with the charged particle:

$$F_a = F_a^s + F_a^r\ ,\qquad F_i = F_i^s + F_i^r + F_i^C \tag{5}$$

The fluxes of charge, momentum and energy are defined as the moments of the velocity distribution functions which must be found from the solution of the kinetic problem.

PARTICLE IN STATIONARY PLASMA

In the case of stationary plasma, the motion of plasma electron or ion in a central force field of the charged particle is determined by the behavior of the effective potential. Under these conditions the plasma specie energy E and angular momentum squared J^2 are constants of motion. Expressions for the number densities of plasma species and fluxes of charge and energy are derived from the kinetic equations with the help of the method of trajectory analysis (20-22).

Electrons

For plasma electrons moving in the repelling field of the negatively charged particle, expressions for charge and energy fluxes can be derived easily and obtain the following form

$$I_e^- = 4\pi R^2 N_{e\infty}(kT_{e\infty} / 2\pi m_e)^{1/2} \exp(e\varphi_f / kT_{e\infty}) \tag{6}$$

$$K_e^- = 4\pi R^2 N_{e\infty}(kT_{e\infty} / 2\pi m_e)^{1/2} \exp(e\varphi_f / kT_{e\infty}) \tag{7}$$

Ions

Analysis of the behavior of plasma ions which are attracted by the charged particle is more complicated. Special approximations for the ion velocity distribution function have been used. They are:

1. Maxwell ions.

$$f_{i\infty} = N_{i\infty}(m_i / 2\pi kT_{i\infty})^{3/2} \exp(-m_i v^2 / 2kT_{i\infty}) \tag{8}$$

2. Monoenergetic ions.

$$f_{i\infty} = [m_i^2 N_{i\infty} / 4\pi(2m_i E_0)^{1/2}]\delta(E - E_0) \tag{9}$$

Here, the ion energy E_0 and temperature $T_{i\infty}$ are related to each other as $E_0 = (4/\pi)T_{i\infty}$.

3. Cold ions.

This approximation represents a special case of Maxwell or monoenergetic ions in the limit $T_{i\infty} \to 0$.

The kinetic problem for plasma with ions which submit to a Maxwell distribution results in a nonlinear integral equation, in contrast to the models of monoenergetic and cold ions for which the problem is reducible to ordinary differential equations for plasma potential. These equations are solved numerically by iterative procedures.

The results of the numerical modeling illustrating the pronouncing effect of Debye screening on plasma-particle charge and heat transfer are presented in (14).

PARTICLE IN FLOWING PLASMA

Hypersonic or Cold Ions Flow Conditions

Another flow regime (23, 24) corresponds to the case when the thermal velocities of ions $\bar{v}_i$ and electrons $\bar{v}_e$ and the speed of plasma relative to the particle V maintain the following relationship between them: $\bar{v}_i << V << \bar{v}_e$. This means that electrons arrive at the particle surface from all directions, whereas ions due to their relatively low thermal velocities are streaming toward the particle in one direction with the speed of plasma flow. The ion trajectories are affected by electrostatic field near the charged particle. Since the thermal velocity of jth kind of plasma species is $\bar{v}_j \sim (kT_{j\infty} / m_j)^{1/2}$, the ion-electron temperature ratio $\tau = T_{i\infty} / T_{e\infty}$ and speed ratio $s = V / (2kT_{e\infty} / m_i)^{1/2}$ are ordered as $\tau^{1/2} << s$. Therefore, the hypersonic ($\mathbf{M} = [2/(1+\gamma\tau)]^{1/2} s >> 1$) plasma flows or two-temperature plasma flows with cold ions ($\tau \to 0$) are considered.

The fluxes of charge, momentum and energy transferred to the particle by heavy plasma species can be expressed in terms of the effective cross sections of direct collisions with the particle (S_h^s), reflection from its surface (S_h^r), and Coulombic interaction (S_i^C) of noncollected ions deflected by the electrostatic field of the charged particle, namely

$$S_h^s = \pi\rho_{mh}^2 , \quad S_h^r = 2\pi \int_0^{\rho_{mh}} \cos\Phi_1 \, \rho d\rho , \quad S_i^C = 2\pi \int_{\rho_{mi}}^{\infty} (1 - \cos\Psi)\rho d\rho \tag{10}$$

The maximum impact parameter ρ_{mh}, the angle of orientation $\Phi_1 = \Phi(v, \rho, R)$ and the scattering angle $\Psi(v, \rho)$ are determined from the solution of the equations of motion of heavy plasma species in the electrostatic field of the charged particle.

The fluxes of heavy plasma species and their energy are calculated as

$$I_h^- = N_{h\infty} V S_h^s , \quad K_h^- = N_{h\infty} V (\tfrac{1}{2} m_h V^2 - Z_h e\varphi_f) S_h^s , \quad K_h^+ = 2kT_s I_h^- \tag{11}$$

Due to a highly negative potential of the particle repelling the greater part of incoming electrons, and negligibly low particle velocity compared to the electron thermal velocity, the plasma electrons submit to Maxwell-Boltzmann distribution. Thus, the macroscopic parameters of electrons (density, fluxes, etc.) can be easily calculated, as described earlier.
Charge and heat transfer to spherical metallic particle under hypersonic flow conditions has been studied by numerical method of direct modeling in (19).

DRAG FORCE

As it was pointed out in (4), the calculations of forces acting on a particle in plasma can be performed using two different methods. The first one (4, 7, 11, 18) is based on a pressure balance at the surface of the body and consists in the determination of the ion momentum distribution over the sphere. The drag contribution due to electrostatic interaction (both for collected and noncollected ions) can not be taken into account with the help of this method, and thus it leads to incorrect results.
In the second method (3, 4, 9, 10, 19), the balance of forces is done through the flight trajectories of ions. This method consists in taking the difference between momentum of each ion before (at infinity for incoming ions) and after (at the particle surface for collected ions and at infinity for outgoing noncollected ions) its interaction with the charged particle.
The reasons of this discordance are the following. During its motion, not only the plasma ion is attracted by the particle, but the charged particle is also attracted by the ion. Therefore, determination of the ion velocity near the surface is not sufficient, and it is necessary to consider all the "history" of the ion-particle interaction.

Subsonic flow

In the case of relatively low speed of plasma flow V, the velocity distribution functions of heavy plasma species in nondisturbed flow region far from the particle can be represented as

$$f_{h\infty} = [1 + 2\mathbf{c}\mathbf{s}_h]f_{h\infty} = f^0_{h\infty} + f^1_{h\infty} \tag{12}$$

where $f_{h\infty} = N_{h\infty}(m_h / 2\pi k T_{h\infty})^{3/2}\exp(-c^2)$, $\mathbf{c} = \mathbf{v}/(2kT_{h\infty}/m_h)^{1/2}$, $\mathbf{s}_h = \mathbf{V}/(2kT_{h\infty}/m_h)^{1/2}$.
The charge fluxes and potential distribution around the particle with an accuracy up to $\sim s_h$ are determined by the symmetrical part $f^0_{h\infty}$ of the velocity distribution function, while the intensity of plasma-particle momentum transfer is conditioned by the anisotropic component $f^1_{h\infty}$.
The drag force components are calculated as

$$F^t_h = (16/3)\pi^{1/2}R^2 N_{h\infty}kT_{h\infty}s_h\psi^t_h\,, \qquad h = a, i; \quad t = s, r, C \tag{13}$$

where

$$\begin{aligned}
\psi_h{}^s &= 2\int_0^\infty\int_0^{\xi_m}\xi d\xi\, c^5\exp(-c^2)dc \\
\psi_h{}^r &= (\pi\tau_s)^{1/2}\int_0^\infty\int_0^{\xi_m}\cos\Phi_1\,\xi d\xi\, c^4\exp(-c^2)dc \\
\psi^C_i &= 2\int_0^\infty\int_{\xi_m}^\infty(1-\cos\Psi)\xi d\xi\, c^5\exp(-c^2)dc
\end{aligned} \tag{14}$$

Here $\xi = \rho/R$, $\xi_m = \rho_{mh}/R$, $\tau_s = T_s/T_{h\infty}$. The maximum impact parameter ρ_{mh}, the angle of orientation $\Phi_1 = \Phi(v,\rho,R)$ and the scattering angle $\Psi(v,\rho)$ are determined by simulation of motion of heavy plasma species in the central force field of the charged particle.

Hypersonic flow

For hypersonic or cold ions flow conditions, the method of calculation described above leads to the following expressions for the drag force components (19)

$$F_h^s = m_h N_{h\infty} V^2 S_h^s, \quad F_h^r = m_h N_{h\infty} (\pi k T_s / 2m_h)^{1/2} V S_h^r, \quad F_i^C = m_i N_{i\infty} V^2 S_i^C \qquad (15)$$

where the effective cross sections are defined by Eq. (10).

As in the case of subsonic flow, the contribution of ions to the total drag force is especially significant for large Debye lengths due to the interaction of ions with attractive field of the charged particle. However, with the growth of plasma flow velocity relative to the particle, ions obtain high kinetic energy of direct motion as compared with the energy of the electrostatic interaction, and thus only direct collisions with the particle become important.

LIMITING CASES

Analytical models of plasma-particle interaction have been used for the description of the two limiting regimes of plasma screening: strong Debye screening (or particle with thin plasma sheath, $r_D << R$) and a weak one (particle with thick sheath, $r_D > R$).

In the case of strong screening, it is usually assumed that electric field exists only in extremely thin sheath surrounding the particle and does not penetrate into the plasma, which enables to simplify the analysis (5-19). Such an approximation may be also called as weak field approximation.

At weak screening, as follows from the analysis of the effective potential of interaction, the cross sections of the collisions of electrons and ions with a particle depend only on the potential of its surface φ_f, but not on the spatial distribution of potential φ in the plasma, which makes it possible to use the known velocity distribution functions in a nondisturbed region of plasma far from the particle in calculation of the fluxes (1, 3, 9, 10, 14, 19). This approximation is also called as orbital regime.

The approximations of strong and weak Debye screening were used for the modeling of various situations, such as heat transfer to spherical (1, 2, 5, 6, 10) and nonspherical (13, 17) metallic and nonmetallic particles in stationary and flowing plasma, plasma action on thermoemitting particle (12, 15, 16), particle drag force (3, 7, 19) and thermophoresis in plasma (10, 11, 18), etc.

Analytical description of charge, momentum and energy transfer based on the approximation of weak screening shows an excellent agreement with the exact numerical results (14) starting from $r_D / R = 1$.

On the contrary, the weak field approximation and numerical solution give a noticeable difference even in the limit $r_D / R \to 0$. For example, in stationary one-temperature argon plasma the dimensionless particle floating potential $y_f = -e\varphi_f / kT_{e\infty}$, fluxes of charge $j_j = I_j^- / 4\pi R^2 N_{e\infty} (kT_{e\infty} / 2\pi m_i)^{1/2}$ and kinetic energy $k_j = K_j^- / 4\pi R^2 N_e k T_{e\infty} (2kT_{e\infty} / \pi m_i)^{1/2}$ obtain the following values: $y_f = 5.60$, $j_e = j_i = k_e = 1.00$, $k_i = 3.80$ according to analytical description, and $y_f = 5.16$, $j_e = j_i = k_e = 1.55$, $k_i = 5.30$ according to numerical method. Thus, the penetration of the electric field of the charged particle into the plasma can not be ignored under conditions of strong Debye screening.

CONCLUSIONS

Kinetic theory analysis of plasma-particle interaction allows to predict a number of important effects, such as particle charging, increase of heat transfer rate due to the contribution of plasma electrons and ions, growth of particle drag in plasma caused by Coulombic interaction with ions, enhancement of heat transfer from plasma to thermoemitting particle, etc. The results of the modeling of plasma-particle interaction show that the effects associated with transfer of charges start manifesting themselves at degrees of gas ionization exceeding several percent.

Among the main parameters which determine the efficiency of plasma action in free-molecular regime are the Debye length to particle radius ratio, electron to ion temperature ratio and speed ratio. Depending on these parameters, special approximations can be applied to the description of charge, heat and momentum transfer.

NOMENCLATURE

e - electron charge, E - energy, E_i - ionization energy, f_j - velocity distribution function, F_h - drag force, $I_j^{\mp}$ - flux of number of plasma species, k - Boltzmann constant, $K_j^{\mp}$ - flux of kinetic energy, Kn - Knudsen number, l_j - mean free path, m_j - mass, M - Mach number, N_j - number density, Q_j - heat flux, r - distance, r_D - Debye length, R - particle radius, s - speed ratio, S_h^t - interaction cross-section, T_j - temperature, v - plasma specie velocity, V - plasma flow velocity, W_j - energy of charge state, γ - specific heat ratio, ρ - impact parameter, φ - potential, φ_f - particle floating potential, Φ - angle of orientation, Φ_e - work function, Ψ - scattering angle.
Subscripts: a - molecules, e - electrons, h - heavy plasma species (molecules and ions), i - ions, s - particle surface, +(-) - direction from (to) the particle, ∞ - nondisturbed plasma region far from the particle.

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